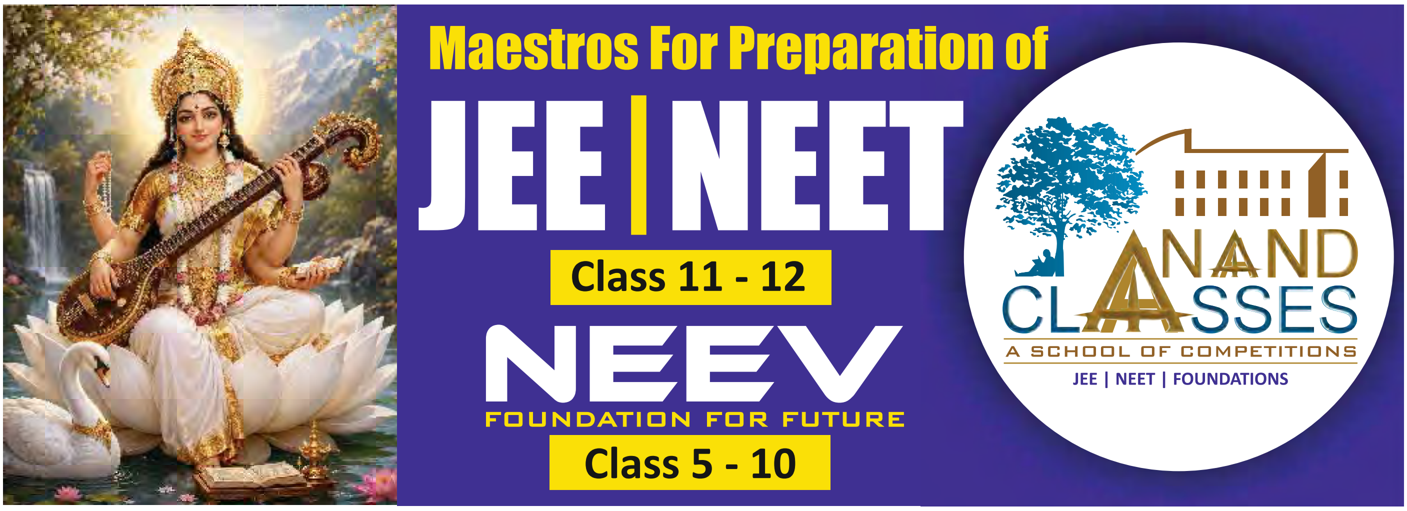

ELECTRIC FIELD LINES OF FORCE

Michael Faraday (1791-1867) introduced the concept of lines of force to visualize the nature of electric (and magnetic) fields. A small positive charge placed in an electric field experiences a force in a definite direction and if it is free to move, it will start moving in that direction. The path along which this charge would move will be a line of force.

The lines of force do not really exist; they are imaginary curves. Yet the concept of lines of force is very useful. Michael Faraday gave simple explanations for many of his discoveries (in electricity and magnetism) in terms of such lines of force. We may define,

> ***An electric field line as a path, straight or curved in electric field, such that tangent to it at any point gives the direction of electric field intensity at that point.***

In fact, it is the path along which a unit positive charge actually moves in the electrostatic field, if free to do so. In Figure AB is an electrostatic line of force. The tangent to the line at any point *P* gives us the direction of electric intensity $\vec{E}_P$ at *P*. Similarly, tangent to AB at *Q* gives us the direction of $\vec{E}_Q$. The field lines are in three-dimensional space, though Figure shows them only in a plane.

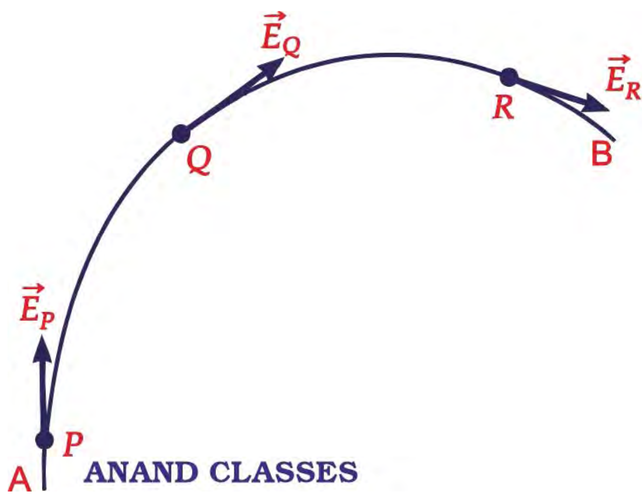

Figure. An electric field line represents the direction of electric field intensity; its tangent at any point gives the direction of the electric field.

PROPERTIES OF ELECTRIC FIELD LINES OF FORCE

1. The electric lines of force are continuous smooth curves without any breaks.

2. The lines of force **start** at **positive charges** and **end** at **negative charges** – they cannot form closed loops. If there is a single charge, then the lines of force will start or end at infinity.

3. ***Tangent to the electric field line at any point gives the direction of electric field intensity at that point.***

4. *No two electric field lines of force can intersect each other.*

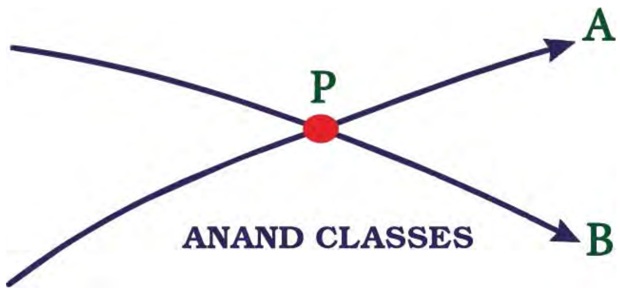

Figure. Two electric field lines cannot intersect because their tangents would indicate two different directions of electric field at the same point.

Reason : This is because at the point of intersection *P*, we can draw two **tangents PA** and **PB** to the two lines of force, Figure. This would mean two directions of electric field intensity at the same point, which is not possible. Hence ***no two electric lines of force can cross each other***.

5. The **electric field lines** are always **normal** to the surface of a conductor on which the charges are in equilibrium.

Reason : If the lines of force are not normal to the conductor, the component of the field $\vec{E}$ parallel to the surface would cause the electrons to move and could set up a current on the surface. But no current flows in the equilibrium condition. Therefore, there is no component of electric field intensity parallel to the surface of the conductor.

6. The lines of force have a tendency to **contract lengthwise**. This explains **attraction** between two unlike charges.

7. The lines of force have a tendency to expand laterally so as to exert a **lateral pressure** on neighbouring lines of force. This explains repulsion between two similar charges.

8. The lines of force **do not** pass through a **conductor** because the electric field inside a charged conductor is zero.

9. Electric field lines pass through dielectrics.

10. The relative closeness of the lines of force gives a measure of the strength of the electric field in any region. The lines of force are
(i) close together in a strong field.
(ii) far apart in a weak field.
(iii) parallel and equally spaced in a uniform field.

Similar concepts include *Electric Field due to Point Charge, Group of Charges, Continuous Charge Distribution Numerical Problems*

ELECTRIC FIELD LINES FOR DIFFERENT CHARGED CONDUCTORS

Electric field lines for different charge systems are explain as follows :

(i) Electric Field Lines of a Positive Point Charge

Figure shows the lines of force of an isolated positive point charge. They are directed radially outwards because a small positive charge would be accelerated in the outward direction. They extend to infinity. The field is **spherically symmetric** i.e., it looks same in all directions, as seen from the point charge.

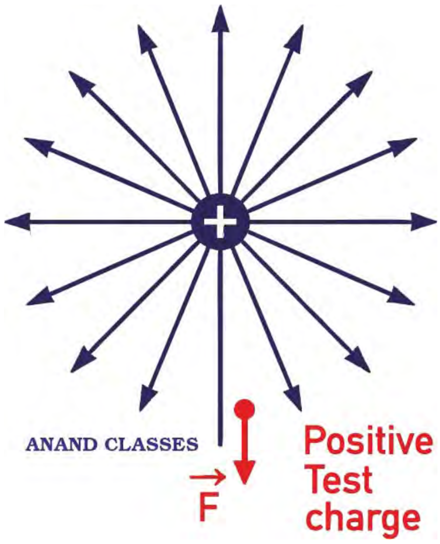

Figure. Electric field lines of an isolated positive point charge are directed radially outward and extend to infinity.

(ii) Electric Field Lines of a Negative Point Charge

The electric field of a negative point charge is also spherically symmetric but the lines of force point radially inwards as shown in Figure. They start from infinity.

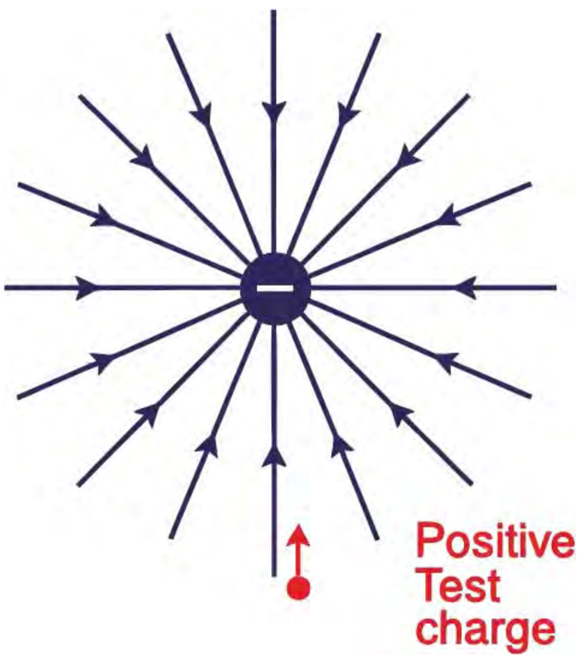

Figure. Electric field lines of an isolated negative point charge are directed radially inward toward the charge.

(iii) Electric Field Lines of Two Equal and Opposite Point Charges (Electric Dipole)

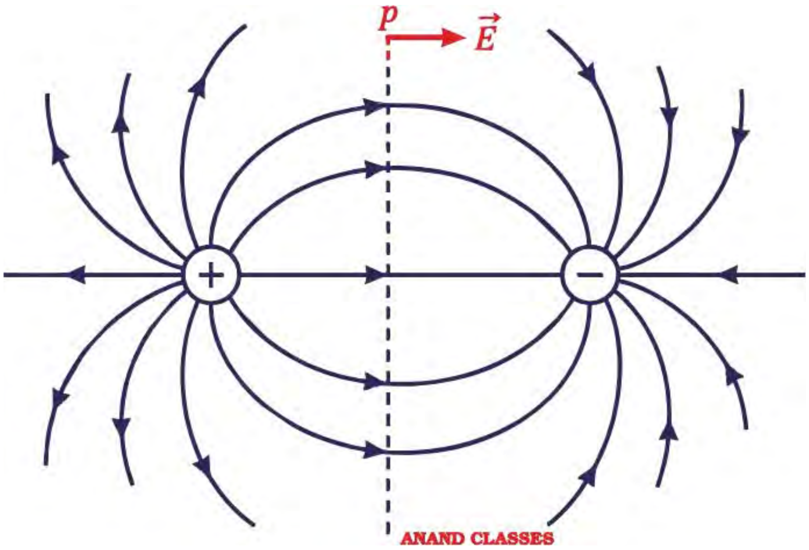

Figure. Electric field lines of an electric dipole start from the positive charge and end on the negative charge.

Figure shows the electric lines of force of an electric dipole i.e., a system of two equal and opposite point charges (+q, −q) separated by a small distance. They start from the positive charge and end on the negative charge. The lines of force seem to contract lengthwise as if the two charges are being pulled together. This explains attraction between two unlike charges. The field is **cylindrically symmetric** about the dipole axis i.e., the field pattern is same in all planes passing through the dipole axis. Clearly, the electric field at all points on the equatorial line is parallel to the axis of the dipole.

(iv) Electric Field Lines of Two Equal and Positive Point Charges

Figure shows the lines of force of two equal and positive point charges. They seem to exert a lateral pressure as if the two charges are being pushed away from each other. This explains repulsion between two like charges. The electric field $\vec{E}$ is zero at the middle point N of the join of two charges. This point is called ***neutral point*** from which no line of force passes. This field also has ***cylindrical symmetry***.

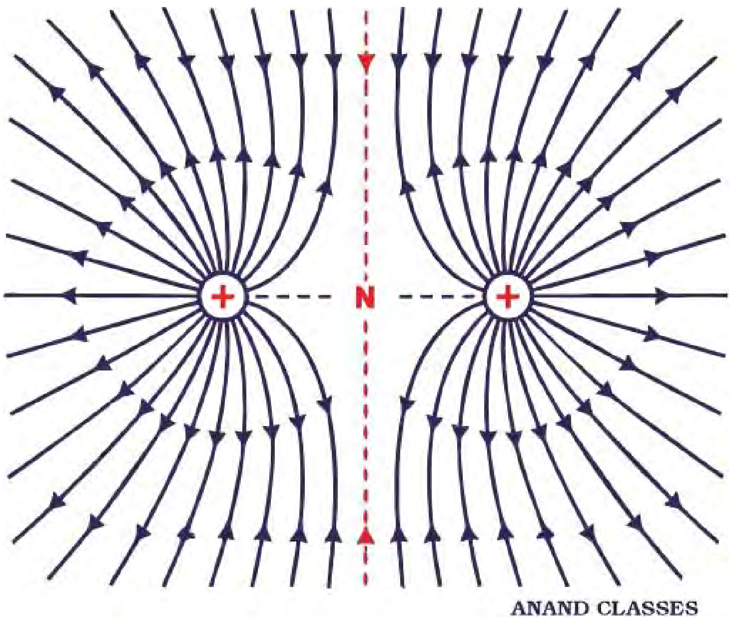

Figure. The field pattern of two equal positive charges shows repulsion and a neutral point midway between the charges.

(v) Electric Field Lines of Two Unequal and Positive Point Charges

Figure shows the lines of force of two unequal and positive point charges. They seem to exert a lateral pressure as if the two charges are being pushed away from each other. This explains repulsion between two like charges.

When the charges are equal, ***neutral point*** *N* lies at the centre of the line joining the charges. However, when the charges are unequal, the ***neutral point N*** is closer to the ***smaller charge*** as shown in Figure.

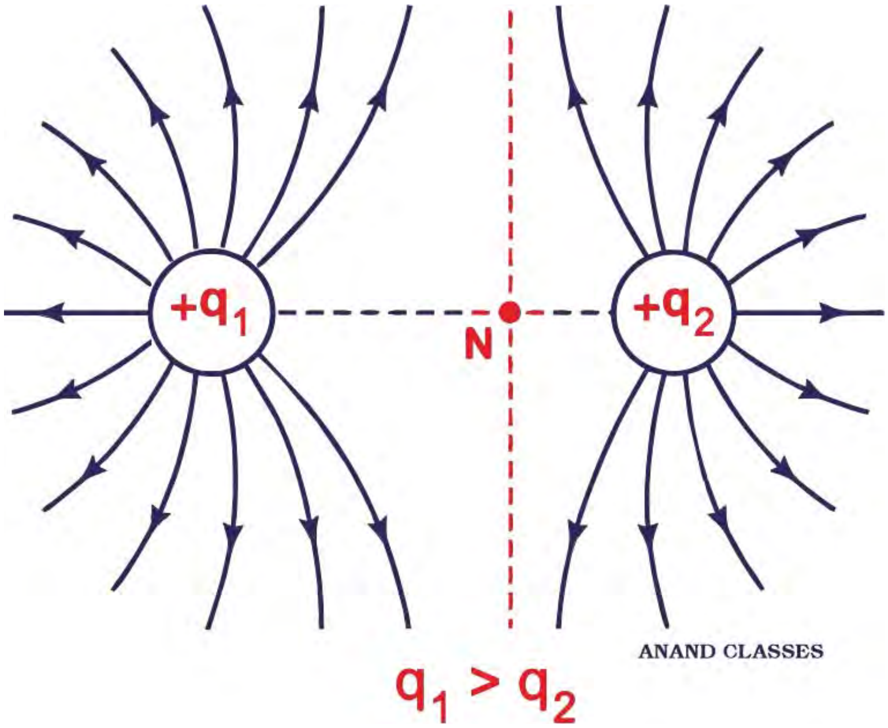

Figure. For two unequal positive charges, the neutral point lies closer to the smaller charge.

(vi) Electric Field Lines of a Positively Charged Plane Conductor

Figure shows the pattern of electric lines of force of positively charged plane conductor. A small positive charge would tend to move normally away from the plane conductor. Thus the lines of force are parallel and normal to the surface of the conductor. They are equispaced, indicating that electric field E is uniform at all points near the plane conductor.

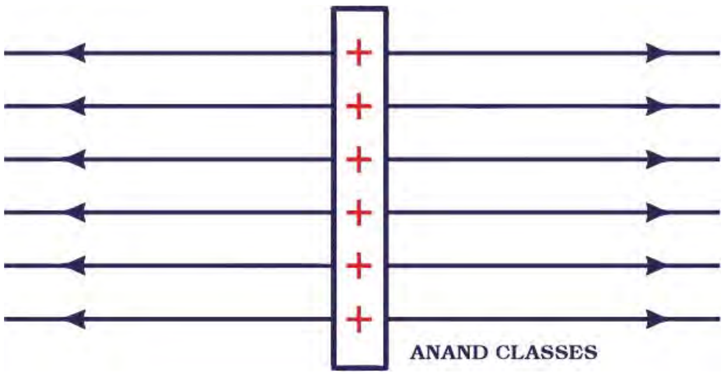

Figure. Electric field lines near a positively charged plane conductor are parallel, equally spaced, and normal to the conductor surface.

Related topics include *[Forces Between Multiple Charges: Principle of Superposition Solved Numerical Problems]*

RELATION BETWEEN ELECTRIC FIELD STRENGTH AND DENSITY OF ELECTRIC LINES OF FORCE

Electric field strength is proportional to the density of lines of force i.e., electric field strength at a point is proportional to the number of lines of force cutting a unit area element placed normal to the field at that point. As illustrated in Figure, the electric field at P is stronger than at Q.

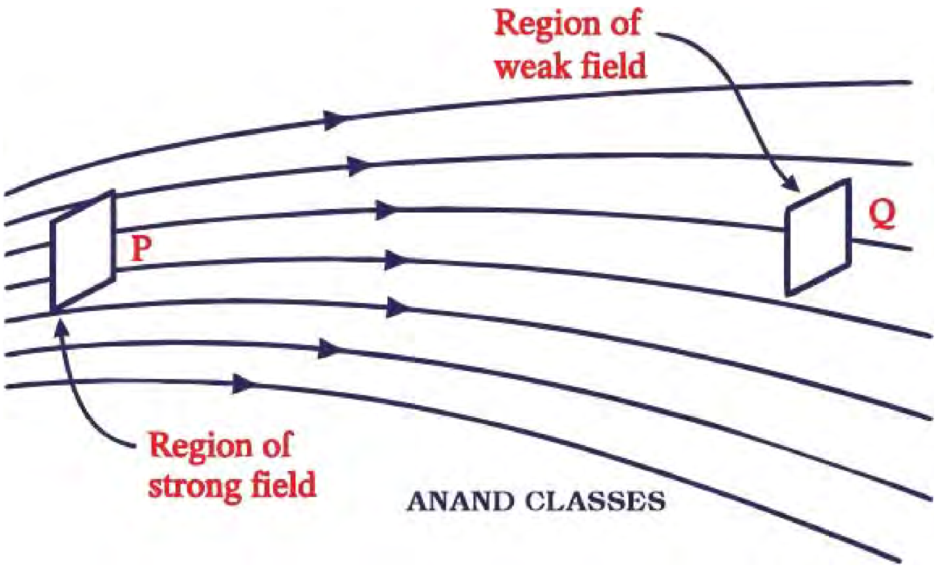

Figure. The density of electric field lines represents the strength of the electric field: closer lines indicate a stronger field.

CONSISTENCY OF THE INVERSE SQUARE LAW WITH THE ELECTRIC FIELD LINES

The magnitude of the field is represented by the density of field lines. Near the charge, lines are closer, so the density of field lines is more and hence electric field $\vec{E}$ is stronger near the charge. Away from the charge, the field lines are well separated, the density of field lines is smaller and hence electric field $\vec{E}$ is weaker, away from the charge. We may, therefore, define

> ***Electric field intensity at a point is equal to number of field lines crossing normally a unit area around that point.***

As field lines are three dimensional and all figures are drawn on the plane of the paper (in two dimensions), therefore, to estimate the density of field lines, we have to consider the number of field lines per unit cross-sectional area perpendicular to the lines. We know that in three dimensions, the solid angle $\Delta\Omega$ subtended by a small perpendicular plane area ΔS at a distance r from apex can be written as $\Delta\Omega = \Delta S/r^2$.

With a charge Q at the apex O of the cone, number of radial field lines in a given solid angle is the same. In Figure, let us consider two points P_1 and P_2 at distances r_1 and r_2 respectively from the charge.

At P_1, area subtended by the solid angle $\Delta\Omega$ is $r_1^2\ .\Delta\Omega$. At P_2, area subtended by the solid angle $\Delta\Omega$ is $r_2^2\ .\Delta\Omega$.

The number of field lines cutting these area elements are the same. Let it be n.

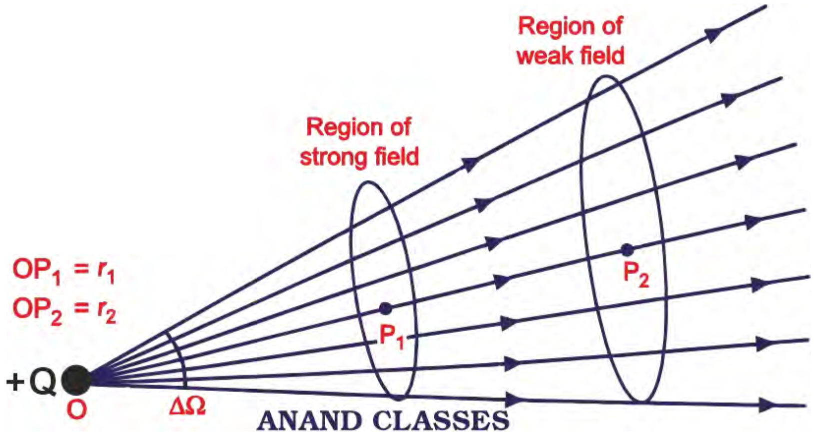

Figure. The increasing separation of field lines with distance illustrates that electric field strength decreases according to the inverse square law.

Therefore number of field lines cutting unit area element at P_1 = Electric field intensity at P_1, i.e., $E_1 = \frac{n}{r_1^2 \cdot \Delta\,\Omega}$ and number of electric field lines cutting unit area element at P_2 = Electric field intensity at P_2, i.e., $E_2 = \frac{n}{r_2^2 \cdot \Delta\,\Omega}$. Clearly;

$$\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}, \text{ i.e., } E \propto \frac{1}{r^2}$$

i.e., electric field intensity is more near the charge and vice-versa.

***Explore more concepts related to** [Coulomb's Law of Electrostatics: Formula, Vector Form, Examples & Numericals]*

ELECTRIC FIELD DUE TO A INFINITELY LONG THIN WIRE OF UNIFORM LINEAR CHARGE DENSITY

Consider an infinite line of charge with uniform line charge density λ as shown in Figure.

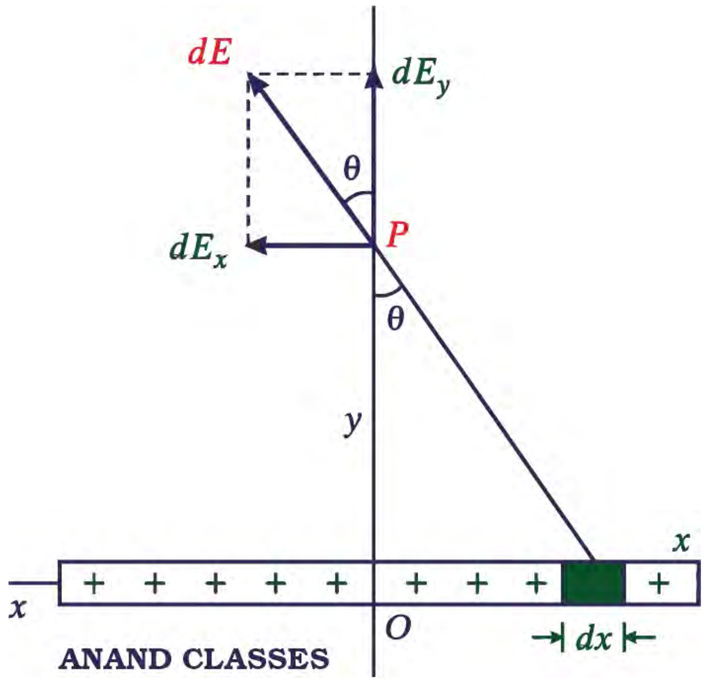

Figure. Electric field at point P due to an infinitely long thin wire with uniform linear charge density λ.

We wish to determine its electric field at any point *P* at a distance *y* from it. The charge on small element *dx* of the line charge will be *dq* = λdx.

The electric field at the point *P* due to the charge element *dq* will be

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{y^2 + x^2}$$

The field $d\vec{E}$ has two components **:**

$dE_x = -dE\sin\theta$ and $dE_y = dE\cos\theta$

The negative sign in x-component indicates that $d\vec{E}_x$ acts in the negative *x*-direction. Every charge element on the right has a corresponding charge element on the left. The *x*-components of two such charge elements will be equal and opposite and hence cancel out. The resultant field $\vec{E}$ gets contributions only from *y*-components and is given by

$$E = E_y = \int_{x=-\infty}^{x=+\infty} d\,E_y = \int_{x=-\infty}^{x=+\infty} \cos\,\theta dE$$

$$E = 2\int_{x=0}^{x=\infty} \cos\,\theta \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{y^2 + x^2}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_{x=0}^{x=\infty} \cos\,\theta \frac{dx}{y^2 + x^2}$$

Now $x = y\tan\theta$, $dx = y\sec^2\theta d\theta$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_{\theta=0}^{\theta=\pi/2} \cos\,\theta \frac{y\sec^2\theta d\theta}{y^2(1+\tan^2\theta)}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\int_{\theta=0}^{\theta=\pi/2} \cos\,\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}[\sin\theta]_0^{\pi/2}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}$$

Students should also study *Electric Charge Quantization, Additivity, Charging by Induction, Solved Numericals*

ELECTRIC FIELD INTENSITY AT ANY POINT ON THE AXIS OF THE UNIFORMLY CHARGED RING

A charge is distributed uniformly over a ring of of **radius '*a*.** Suppose that the ring is placed with its plane perpendicular to the *x*-axis, as shown in Figure.

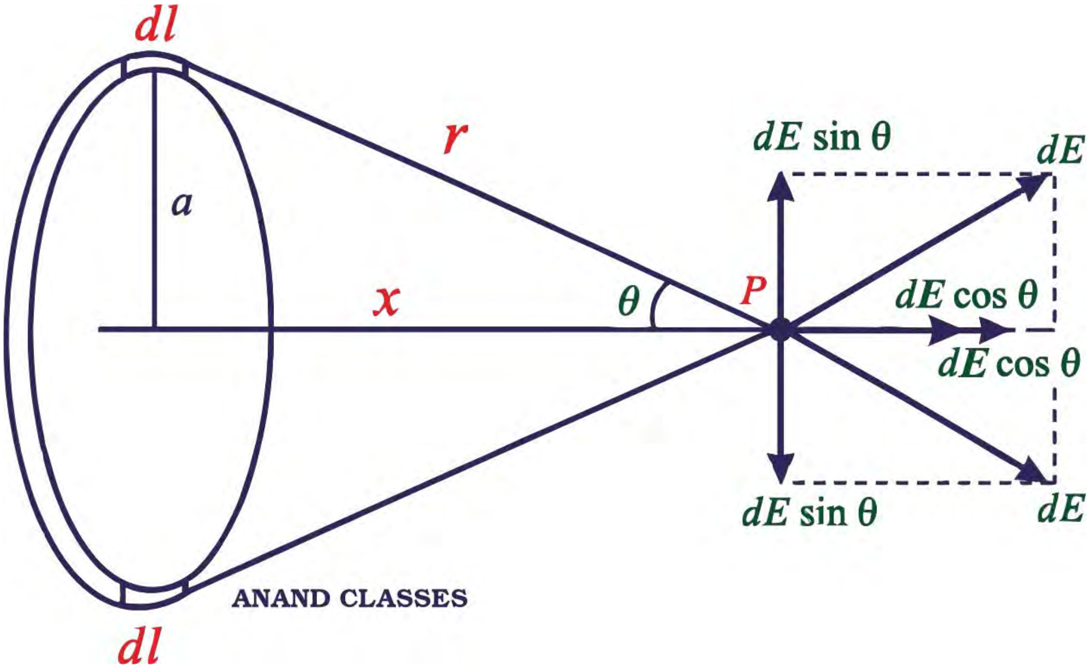

Figure. The electric field at an axial point of a uniformly charged ring is obtained by adding the axial components of the fields produced by its charge elements.

Consider a small element *dl* of the ring. As the **total charge *q*** is uniformly distributed, the charge *dq* on the element *dl* is

$$dq = \frac{q}{2\pi a} \cdot dl$$

Therefore the magnitude of the field $d\vec{E}$ produced by the element *dl* at the field point *P* is

$$dE = k \cdot \frac{dq}{r^2} = \frac{kq}{2\pi a} \cdot \frac{dl}{r^2}$$

As shown in Figure, the field $d\vec{E}$ has two components **:**

The axial component *dE* cosθ, and the perpendicular component *dE* sinθ.

Since the perpendicular components of any two diametrically opposite elements are equal and opposite, they all cancel out in pairs. Only the axial components will add up to produce the resultant field *E* at point *P*, which is given by

$$E = \int_0^{2\pi a} d\,E \cos\theta$$

because only the axial components contribute towards *E*

$$E = \int_0^{2\pi a} \frac{kq}{2\pi a} \cdot \frac{dl}{r^2} \cdot \frac{x}{r} = \frac{kqx}{2\pi a r^3} \int_0^{2\pi a} d\,l$$

$$E = \frac{kqx}{2\pi a r^3} \cdot 2\pi a$$

$$E = \frac{kqx}{r^3}$$

because cosθ = *x*/*r* and $r = \sqrt{x^2 + a^2}$

$$E = \frac{kqx}{(x^2 + a^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(x^2 + a^2)^{3/2}}$$

Special case : For points at large distances from the ring, ***x* >> *a***

$$E = \frac{kq}{x^2} = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$$

This is the same as the field due to a point charge, indicating that for far off axial points, the charged ring behaves as a point charge.

Note : At the centre of the uniformly charged ring, the electric field is zero. The contributions from opposite elements cancel completely at the centre.

At center of the ring, $x = 0$ and $E = 0$.

ELECTRIC FIELD INTENSITY AT THE CENTER OF THE UNIFORMLY CHARGED SEMICIRCULAR RING

A thin semicircular ring of radius a is charged uniformly and the charge per unit length is λ. As the **total charge *q*** is uniformly distributed, the charge dq on the element dl is

$$dq = \lambda\, dl$$

Consider two symmetric elements each of length dl at A and B. The electric fields of the two elements perpendicular to PO get cancelled while those along PO get added.

Electric field at center O due to an element of length dl is

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{a^2} \cos\theta$$

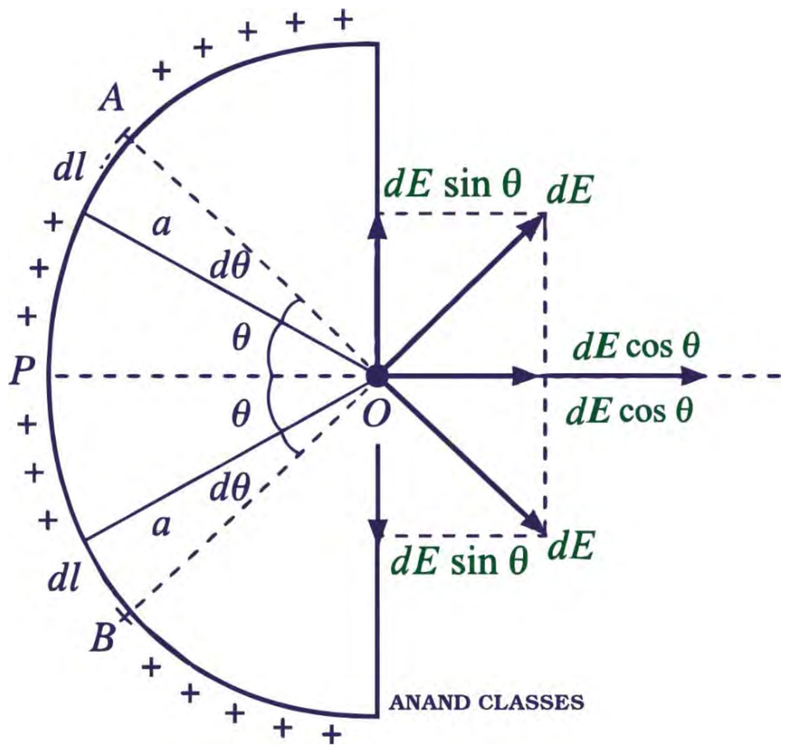

Figure. For a uniformly charged semicircular ring, perpendicular field components cancel while the components along the symmetry axis add.

The direction of *dE* is along *PO*.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{a^2}\cos\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(ad\theta)}{a^2}\cos\theta$$

[Since, $dl = a\, d\theta$]

Total electric field at the centre *O* is

$$E = \int_{-\pi/2}^{\pi/2} d\,E = 2\int_{0}^{\pi/2}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\cos\theta d\theta}{a}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}[\sin\theta]_0^{\pi/2} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a}\cdot 1 = \frac{\lambda}{2\pi\varepsilon_0 a}$$

🕮 SHORT CONCEPTUAL QUESTIONS AND ANSWERS ON ELECTRIC FIELD LINES FOR CLASS 11 PHYSICS, JEE, NEET

🕮 Why should a test charge be of negligibly small magnitude?

The magnitude of the test charge must be small enough so that it does not disturb the distribution of the charges whose electric field we wish to measure otherwise the measured field will be different from the actual field.

🕮 In defining electric field due to a point charge, the test charge has to be vanishingly small. How this condition can be justified, when we know that charge less than that on an electron or a proton is not possible?

Because of charge quantisation, the test charge qo cannot go below e. However, in macroscopic situations, the source charge is much larger than the charge on an electron or proton, so the limit $q_0 \to 0$ for the test charge is justified.

🕮 What is the advantage of introducing the concept of electric field ?

By knowing the electrical field at a point, the force on a charge placed at that point can be determined.

🕮 How do charges interact?

The electric field of one charge exerts a force on the other charge and vice versa.

🕮 An electron and a proton are kept in the same electric field. Will they experience same force and have same acceleration ?

Both electron and proton will experience force of same magnitude, $F = eE$. Since a proton has 1836 times more mass than an electron, so its acceleration will be 1/1836times that of the electron.

🕮 Why direction of an electric field is taken outward (away) for a positive charge and inward (towards) for a negative charge?

By convention, the direction of electric field is the same as that of force on a unit positive charge. As this force is outward in the field of a positive charge, and inward in the field of a negative charge, so the directions are taken accordingly.

🕮 A charged particle is free to move in an electric field. Will it always move along an electric field ? [IIT]

The tangent at any point to the line of force gives the direction of electric field and hence of force on a charge at that point. If the charged particle starts from rest, it will move along the line of force. If it is in motion and moves initially at an angle with the line of force, then resultant path is not along the line of force.

🕮 A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force? [NCERT]

Not necessarily. The test charge will move along the line of force only if it is a straight line. This is because a line of force gives the direction of acceleration and not that of velocity.

🕮 Why do charges reside on the surface of the conductor?

Charges lie at the ends of lines of force. These lines of force have a tendency to contract in length. The lines of force pull charges from inside a conductor to its outer surface.

🕮 Why is electric field zero inside a charged conductor?

This is because charges reside on the surface of a conductor and not inside it.

🕮 Why do the electrostatic field lines not form closed loops?

Electrostatic field lines start from a positive charge and end on a negative charge or they fade out at infinity in case of isolated charges without forming any closed loop.

Alternatively, electrostatic field is a conservative field. The work done in moving a charge along a closed path must be zero. Hence, electrostatic field lines cannot form closed loops.

🕮 Do the electric lines of force really exist? What is about the field they represent?

Lines of force do not really exist. These are hypothetical curves used to represent an electric field. But the electric field which they represent is real.

🕮 Draw lines of force to represent a uniform electric field.

The lines of force of a uniform electric field are equidistant parallel lines as shown in Figure.

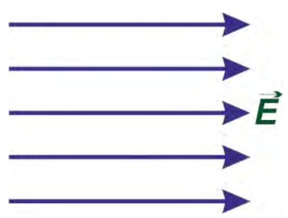

Figure. A uniform electric field is represented by parallel, equally spaced electric field lines.

🕮 A positive point charge (+q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point charge on to the surface of the plate.

Starting from the charge +q, the lines of force will terminate at the metal plate, inducing negative charge on it. At all positions, the lines of force will be perpendicular to the metal surface.

🕮 Why is it necessary that the field lines from a point charge placed in the vicinity of a conductor must be normal to the conductor at every point.

If the field lines are not normal, then the electric field would have a tangential component which will make electrons move along the surface creating surface currents and the conductor will not be in equilibrium.

🕮 The force on an electron kept in an electric field in a particular direction is F. What will be the magnitude and direction of the force experienced by a proton at the same point in the field? Mass of the proton is 1836 times the mass of the electron.

A proton has charge equal and opposite to that of an electron. Hence the proton will experience a force equal and opposite to that of F.